AP Calculus AB Name: ______________________________

Mr. Gábor Summer Review Assignment Pd. _______

Welcome to AP Calculus AB! This will be your pinnacle math course at CCA, and I want it to be a successful year for you. Heading into the course, there are many math skills that you have learned over the past few years that will be critical for you to know and recall while working through calculus problems. This packet has been designed for you to review/relearn/learn those topics so that you will be ready to learn calculus and to do well on the AP exam.

Do not fake your way through these problems! You will need to understand everything in this packet very well. If you do not fully understand these problems, it is very possible that you will get calculus problems wrong – not because you do not understand the calculus concepts, but because you do not understand the algebra or trigonometry necessary to work the problems to completion.

Show all your work either on the review packet itself or on another sheet of paper. Also, do not rely on your calculator too heavily. Some topics, such as logarithms, lend themselves to the use of a calculator, but others, such as factoring or analytic trigonometry, are ones you should approach without a calculator. Remember: two thirds of the AP exam does not allow any calculator usage.

Additional help: if you find yourself struggling on any particular section of problems, searching the web for that specific topic will yield a myriad of sites that can help you. There are many math instructors who have posted videos on youtube.com that can be of immense help as well.

This worksheet will be due on the first day of class in August.

Good luck with these problems, and most importantly: have a safe and pleasant summer. I look forward to seeing you in August!

AP Calculus Summer Assignment Part 1 of 3: Prerequisites for Calculus

Name:____ **Due Date: June 25th, 2014**

◆ Skill A Writing an equation of a line in slope-intercept form

Recall The slope-intercept form of a line is $y = mx + b$.

$$y = \underset{\text{slope}}{\underset{\uparrow}{m}}x + \underset{y\text{-intercept}}{\underset{\uparrow}{b}}$$

◆ **Example**

Write an equation for each line.

a. containing (0, 1) and with a slope of −2

b. containing (3, −4) and (9, 0)

◆ **Solution**

a. The slope, m, is given as −2. The line contains (0, 1), so this point is the y-intercept, or b is 1. Substituting these numbers into the equation gives $y = -2x + 1$.

b. First find the slope. $m = \frac{-4 - 0}{3 - 9} = \frac{-4}{-6} = \frac{2}{3}$

Then subtitute the coordinates of one of the given points into the equation and solve for b.

For the point (9, 0): $0 = \frac{2}{3}(9) + b$

$0 = 6 + b$

$b = -6$

Substituting this number for b and $\frac{2}{3}$ for m into the equation $y = mx + b$ gives the equation $y = \frac{2}{3}x - 6$.

For each equation, find the slope and the y-intercept.

1. $y = 3x - 1$ ____________ **2.** $y = \frac{1}{2}x + 2$ ____________ **3.** $y = -x + \frac{1}{2}$ ____________

Write an equation in slope-intercept form for each line.

4. with a slope of 2 and a y-intercept of −1 ________________________

5. containing (0, −3) and with a slope of $\frac{1}{3}$ ________________________

Write an equation in slope-intercept form for the line that contains each pair of points.

6. (1, 1) and (3, 5) ____________ **7.** (2, −4) and (−1, 5) ____________

8. (2, 4) and (−4, 1) ____________ **9.** (1, 0) and (3, 2) ____________

◆ Skill B Writing an equation of a line in point-slope form

Recall The point-slope form for an equation of a line is $y - y_1 = m(x - x_1)$.

◆ **Example**

Write an equation for the line through $(1, -1)$ and $(-1, 5)$

a. in point-slope form.

b. in slope-intercept form.

◆ **Solution**

a. First find m.

$$m = \frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}} = \frac{-1 - 5}{1 - (-1)} = \frac{-6}{2} = -3$$

Substitute the slope and one of the points into the point-slope equation.

$y - y_1 = m(x - x_1)$

$y - (-1) = -3(x - 1)$ Use the point $(1, -1)$.

$y + 1 = -3(x - 1)$ Simplify.

b. Rewrite the equation in the form $y = mx + b$.

$y + 1 = -3(x - 1)$

$y + 1 = -3x + 3$ Distributive Property

$y = -3x + 2$ Subtract 1 from each side.

Write an equation for each line in point-slope form.

1. containing $(4, -1)$ and with a slope of $\frac{1}{2}$ ______________________

2. crossing the x-axis at $x = -3$ and the y-axis at $y = 6$ ______________________

3. containing the points $(-6, -1)$ and $(3, 2)$ ______________________

Rewrite each equation in slope-intercept form.

4. the line from Exercise **1.** ______________________

5. the line from Exercise **2.** ______________________

6. the line from Exercise **3.** ______________________

7. In what situations would you find it easier to use point-slope form, and in what situations would you find it easier to use slope-intercept form? ______________________

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◆Skill C Factoring trinomials by choosing factor pairs of the constant

Recall Another way to factor a trinomial, such as $x^2 - 5x - 6$, is to first make a list of the pairs of factors of the constant. Then choose the right combination to complete the factors of the trinomial.

◆ **Example**

Use the constant's factor pairs to factor $x^2 - 5x - 6$.

◆ **Solution**

List each pair of factors of −6 along with their sum.

Factors of −6	Sum of the factors
6 and −1	5
3 and −2	1
2 and −3	−1
1 and −6	−5

The sum of 1 and −6 is −5. Use the combination of 1 and −6 to form the factors.
Thus, $x^2 - 5x - 6 = (x + 1)(x - 6)$.

Factor each trinomial. If the trinomial cannot be factored, write *prime*.

1. $x^2 - x - 2$ ____________

2. $x^2 + 3x - 4$ ____________

3. $x^2 + 4x + 3$ ____________

4. $x^2 - 4x + 3$ ____________

5. $x^2 + 2x - 8$ ____________

6. $x^2 + x - 20$ ____________

7. $x^2 + 2x - 15$ ____________

8. $x^2 - 3x + 10$ ____________

9. $x^2 - x - 12$ ____________

10. $x^2 + 6x + 8$ ____________

11. $x^2 - 20x + 36$ ____________

12. $x^2 + 2x - 24$ ____________

◆Skill D Find the zeros of a polynomial function by factoring

Recall The zeros of a function are the values of x that make y equal to 0.

◆ **Example 1**

Find the zeros of the function $y = (x - 2)(x + 5)$.

◆ **Solution**

Let $y = 0$. Then use the Zero-Product Property to solve for x.

$(x - 2)(x + 5) = 0$

$(x - 2) = 0$ or $(x + 5) = 0$

$x = 2$ or $x = -5$

The zeros of $y = (x - 2)(x + 5)$ are 2 and -5.

Recall A quadratic polynomial can be factored into two binomials.

◆ **Example 2**

Solve the equation $x^2 - x - 6 = 0$.

◆ **Solution**

Since $x^2 - x - 6$ can be factored into $(x + 2)(x - 3)$, you can rewrite $x^2 - x - 6 = 0$ as $(x + 2)(x - 3) = 0$. Solve the equation $(x + 2)(x - 3) = 0$.

$x + 2 = 0$ or $x - 3 = 0$

$x = -2$ or $x = 3$

The solutions to $x^2 - x - 6 = 0$ are -2 and 3.

Solve by factoring.

1. $x^2 - 4x - 12 = 0$

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2. $x^2 - 6x + 9 = 0$

____

3. $x^2 - 9x + 14 = 0$

____

4. $x^2 + 6x + 5 = 0$

____

5. $x^2 - 7x + 10 = 0$

____

6. $x^2 - 36 = 0$

____

7. $x^2 + 8x + 16 = 0$

____

8. $x^2 - x - 12 = 0$

____

9. $9x^2 - 1 = 0$

____

10. $4x^2 + 4x + 1 = 0$

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◆ Skill E Using the quadratic formula to solve equations

Recall The solutions for a quadratic equation written in the form $ax^2 + bx + c = 0$, where $a \neq 0$, can be found by using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

◆ **Example**

Use the quadratic formula to solve $x^2 - 8x + 15 = 0$ for x.

◆ **Solution**

For $x^2 - 8x + 15 = 0$, a is 1; b is -8, and c is 15. Substitute these values in the quadratic formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - (4)(1)(15)}}{(2)(1)}$$

$$= \frac{8 \pm \sqrt{64 - 60}}{2}$$

$$= \frac{8 \pm \sqrt{4}}{2}$$

$$= \frac{8 \pm 2}{2}$$

$x = 3$ or $x = 5$

The solutions are 3 and 5.

Use the quadratic formula to solve each equation.

1. $x^2 - 5x + 4 = 0$

2. $x^2 - 2x - 24 = 0$

3. $x^2 + 6x + 9 = 0$

______ ______ ______

4. $x^2 + 3x - 10 = 0$

5. $2x^2 - x - 6 = 0$

6. $2x^2 + x - 4 = 0$

______ ______ ______

◆Skill F Using the quadratic formula to find the zeros of quadratic functions

Recall The zeros of a quadratic function written in the form $y = ax^2 + bx + c = 0$, where $a \neq 0$, can be found by using the quadratic formula.

◆ **Example**

Use the quadratic formula to find the zeros of $y = 2x^2 - 5x - 3$.

◆ **Solution**

For $2x^2 - 5x - 1$, a is 2; b is -5, and c is -3. Substitute these values in the quadratic formula.

$$\frac{-(-5) \pm \sqrt{(-5)^2 - (4)(2)(-3)}}{(2)(2)} = \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5 \pm \sqrt{49}}{4} = \frac{5 \pm 7}{4}$$

The zeros are $-\frac{1}{2}$ and 3.

Use the quadratic formula to find the zeros of each function.

1. $y = x^2 + 2x - 8$ **2.** $y = 2x^2 - x - 15$ **3.** $y = 4x^2 - 8x + 3$

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◆Skill G Using the discriminant to determine the number of solutions

Recall When a quadratic equation is written in the form $ax^2 + bx + c = 0$, where $a \neq 0$, the expression $b^2 - 4ac$ is called the *discriminant* of the quadratic formula.
If $b^2 - 4ac > 0$, there are two solutions.
If $b^2 - 4ac = 0$, there is one solution.
If $b^2 - 4ac < 0$, there are no real number solutions.

◆ **Example**

What does the discriminant tell you about $3x^2 - 2x + 9 = 0$?

◆ **Solution**

For $3x^2 - 2x + 9 = 0$, a is 3; b is -2, and c is 9.
Thus, $b^2 - 4ac = (-2)^2 - (4)(3)(9) = 4 - 108 = -104$
$-104 < 0$, so the equation $3x^2 - 2x + 9 = 0$ has no real solutions.

Give the value of each discriminant. What does the discrimant tell you about the function?

1. $y = 4x^2 + 4x + 1$ **2.** $y = x^2 + 5x + 4$ **3.** $y = x^2 + 5x + 8$

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◆ Skill H Writing and evaluating functions

Recall The value of $f(x) = x^2 + 5$ depends on the value of x.

◆ **Example 1**

Sarah uses an internet server which charges $12.50 per month plus $0.60 for each hour over 20 hours that she uses it during the month. Write this relation in function notation. How much will she be charged for using the service for 38 hours in April?

◆ **Solution**

Let h = number of hours over 20. Thus, the function is as follows.

$f(h) = 12.50 + 0.60h$

$f(18) = 12.50 + 0.60(18)$ *where h = 18*

$f(18) = 23.30$

The charge for April will be $35.30.

◆ **Example 2**

If $g(x) = x^2 + 3x$, find $g(-5)$.

◆ **Solution**

$g(-5)$ means replace x with the value -5 and evaluate $g(x)$.

$$g(-5) = (-5)^2 + 3(-5) = 25 - 15 = 10$$

Thus, $g(-5) = 10$.

Let $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$. Evaluate each function.

1. $f(6)$ ______

2. $f(0)$ ______

3. $f\left(\frac{1}{2}\right)$ ______

4. $g(1)$ ______

5. $g(-2)$ ______

6. $g\left(\frac{1}{2}\right)$ ______

7. $f(1) + g(0)$ ______

8. $g(4) - f(5)$ ______

9. $f(0) \cdot g(0)$ ______

10. $g(-6) \cdot f(-6)$ ______

◆Skill | Using the four basic operations on functions to write new functions

Recall To write the sum, difference, product, or quotient of two functions, f and g, write the sum, difference, product, or quotient of the expressions that define f and g. Then simplify.

◆ **Example**

Let $f(x) = x^2 + 3x + 2$ and $g(x) = 5x - 1$. Write an expression for each function.

a. $(f + g)(x)$ **b.** $(f - g)(x)$ **c.** $(fg)(x)$ **d.** $\left(\frac{f}{g}\right)(x)$

◆ **Solution**

a. $(f + g)(x) = f(x) + g(x)$

$= (x^2 + 3x + 2) + (5x - 1)$

$= x^2 + 8x + 1$ *Combine like terms.*

b. $(f - g)(x) = f(x) - g(x)$

$= (x^2 + 3x + 2) - (5x - 1)$

$= x^2 + 3x + 2 - 5x + 1$

$= x^2 - 2x + 3$ *Combine like terms.*

c. $(fg)(x) = f(x) \cdot g(x)$

$= (x^2 + 3x + 2)(5x - 1)$

$= (x^2 + 3x + 2)(5x) + (x^2 + 3x + 2)(-1)$ *Distributive Property*

$= 5x^3 + 15x^2 + 10x - x^2 - 3x - 2$

$= 5x^3 + 14x^2 + 7x - 2$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

$= \frac{x^2 + 3x + 2}{5x - 1}$, where $x \neq \frac{1}{5}$

Let $f(x) = 3x^2 + 2$, $g(x) = 2x - 1$, and $h(x) = x^2 + 5x$. Find each new function, and state any domain restrictions.

1. $(f + g)(x)$

____

2. $(f - h)(x)$

____

3. $(h - g)(x)$

____

4. $(gh)(x)$

____

5. $(hg)(x)$

____

6. $(f + h)(x)$

____

7. $\left(\frac{f}{g}\right)(x)$

____

8. $\left(\frac{h}{g}\right)(x)$

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◆Skill J Finding the composite of two functions

Recall To write an expression for the composite function $(f \circ g)(x)$, replace each x in the expression for f with the expression defining g. Then simplify the result.

◆ **Example**

Let $f(x) = 5x$ and $g(x) = 2x^2 - 3$. Find $(f \circ g)(2)$ and $(g \circ f)(2)$. Then write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$.

◆ **Solution**

$(f \circ g)(2)$: $g(2) = 2(2)^2 - 3 = 5$ $f(g(2)) = f(5) = 5(5) = 25$

Thus, $(f \circ g)(2) = 25$.

$(g \circ f)(2)$: $f(2) = 5(2) = 10$ $g(f(2)) = g(10) = 2(10)^2 - 3 = 197$

Thus, $(g \circ f)(2) = 197$.

To write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$, use the variable x instead of a particular number.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x^2 - 3) \\ &= 5(2x^2 - 3) \\ &= 10x^2 - 15\end{aligned} \qquad \begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(5x) \\ &= 2(5x)^2 - 3 \\ &= 50x^2 - 3\end{aligned}$$

Let $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$. Find each composite function.

1. $(f \circ g)(x)$ ________________

2. $(g \circ f)(x)$ ________________

3. $(h \circ f)(x)$ ________________

4. $(h \circ g)(x)$ ________________

5. $(g \circ g)(x)$ ________________

6. $(h \circ h)(x)$ ________________

7. $(g \circ h)(4)$ ________________

8. $(f \circ f)(-3)$ ________________

9. $(f \circ (g \circ h))(1)$ ________________

10. $(g \circ (g \circ g))(5)$ ________________

◆Skill K Evaluating and applying rounding-up, rounding-down, and absolute value functions

Recall The rounding-up function rounds a decimal value to the next highest integer.
$\lceil -2.3 \rceil = -2$

◆ **Example 1**

A long distance telephone company advertises that weekend calls cost $0.10 per minute. Each fraction of a minute is rounded up to the next whole minute. Write this as a rounding-up function. Then find the cost of a 23.5-minute call.

◆ **Solution**

$f(m) = 0.1\lceil m \rceil$ *where m = number of minutes*

$f(23.5) = 0.1\lceil 23.5 \rceil$

$= 0.1(24)$ *24 is the next highest integer after 23.5*

$= 2.40$

The call will cost $2.40.

Recall The rounding-down function rounds a decimal value to the next lowest integer.
$[-2.3] = -3$

◆ **Example 2**

Evaluate $[-3.7]$.

◆ **Solution**

$[-3.7] = -4$ *−4 is the next integer to the left of −3.7*

Recall Absolute value means distance from 0. $|-2.3| = 2.3$

◆ **Example 3**

Evaluate $|-12.3|$.

◆ **Solution**

$|-12.3| = 12.3$ *−12.3 is at a distance 12.3 units from 0*

Evaluate.

1. $\|4.2\|$ ________	**2.** $[4.2]$ ________	**3.** $\lceil 4.2 \rceil$ ________
4. $\|-1.8\|$ ________	**5.** $[-1.8]$ ________	**6.** $\lceil -1.8 \rceil$ ________
7. $\|2\| + \|-7\|$ ________	**8.** $\|2\| - \|-7\|$ ________	**9.** $[-2.3] + [-1.8]$ ________
10. $\lceil 2.7 \rceil - \lceil -3.4 \rceil$ ________	**11.** $\lceil -3 \rceil - [-3]$ ________	**12.** $\|-8.5\| + [3.7]$ ________

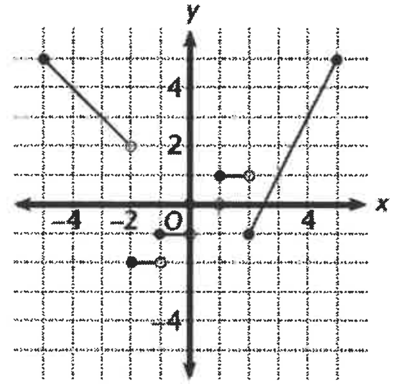
◆Skill L Graphing piecewise, step, and absolute-value functions

Recall A piecewise function in x is a function defined by different expressions in x on different intervals for x.

◆ **Example**

Graph this piecewise function.

$$f(x) = \begin{cases} |x|, & \text{if } -5 \le x < -2 \\ [x], & \text{if } -2 \le x < 2 \\ 2x - 5, & \text{if } 2 \le x \le 5 \end{cases}$$



◆ **Solution**

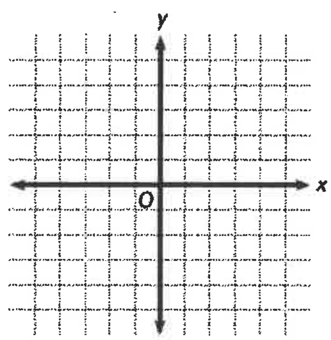
x	-5	-4	-3	-2.5
$y = \|x\|$	5	4	3	2.5

x	-2	-1.5	-1	-0.5	0	1
$y = [x]$	-2	-2	-1	-1	0	1

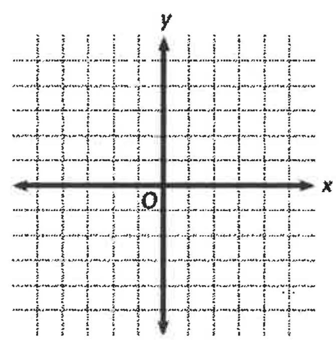
x	2	2.5	3	4	5
$y = 2x - 5$	2	0	1	3	5

Graph each function.

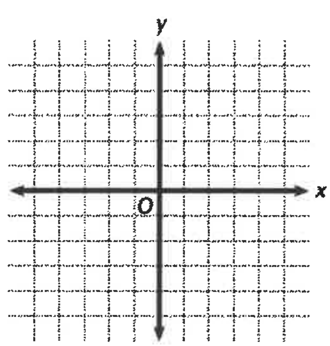
1. $f(x) = \begin{cases} x + 3, \text{ if } x < 0 \\ -2x + 5 \text{ if } x \ge 0 \end{cases}$



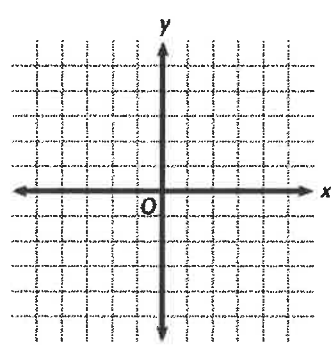
2. $f(x) = \begin{cases} \frac{1}{2}x \text{ if } -4 \le x \le 2 \\ 2x - 3 \text{ if } x > 2 \end{cases}$



3. $f(x) = \begin{cases} |x| \text{ if } x \le 1 \\ 2 - |x - 2| \text{ if } x > 1 \end{cases}$



4. $f(x) = \begin{cases} [x] \text{ if } -2 \le x \le 1 \\ \lceil x \rceil \text{ if } 1 < x \le 4 \end{cases}$



◆Skill M Using logarithms to solve exponential equations

Recall The common logarithm, $\log_{10} x$, is usually written as $\log x$.

◆ **Example**

Solve each equation.

a. $3^x = 81$ **b.** $5^x = 75$ **c.** $7^{x+1} = 150$

◆ **Solution**

a. $3^x = 81$

Since 81 is a power of 3, use powers of 3.

$3^x = 3^4$

$x = 4$ *One-to-One Property of Exponential Functions*

b. $5^x = 75$

Since 75 is **not** a power of 5, use logarithms to solve this equation.

$\log 5^x = \log 75$

$x \log 5 = \log 75$ *Power Property of Logarithms*

$x = \dfrac{\log 75}{\log 5}$

$x \approx 2.68$

Check: $5^{2.68} \approx 75$

c. $7^{x+1} = 150$

$\log 7^{x+1} = \log 150$

$(x + 1)\log 7 = \log 150$

$x + 1 = \dfrac{\log 150}{\log 7}$

$x = \dfrac{\log 150}{\log 7} - 1$

$x \approx 1.57$

Solve each equation. Round your answers to the nearest hundredth.

1. $7^x = 80$ ________

2. $5^x = 10$ ________

3. $6^x = 1296$ ________

4. $4^{x+1} = 100$ ________

5. $2^{x-3} = 25$ ________

6. $3^{x+4} = 27$ ________

7. $6^{2x-7} = 216$ ________

8. $5^{3x-1} = 49$ ________

9. $10^{x+5} = 125$ ________

◆Skill N Finding logarithms with bases other than 10
(You will need a calculator.)

Recall The equation $x = \log_b y$ is equivalent to $b^x = y$.

◆ **Example 1**
Find $\log_3 40$ using your calculator.

◆ **Solution**
Since calculators do not work in base 3, you can change this problem to a base 10 logarithm problem.

$x = \log_3 40 \rightarrow 3^x = 40$

$\log 3^x = \log 40$ *base 10 logarithms*

$x \log 3 = \log 40$

$$x = \frac{\log 40}{\log 3}$$

$x \approx 3.36$ *Use a calculator.*

Recall The change-of-base formula is $\log_b x = \frac{\log_a x}{\log_a b}$, where a can be any permissible logarithmic base.

◆ **Example 2**
Find $\log_5 68$ using your calculator.

◆ **Solution**
To find $\log_5 68$, use logarithms with base 10. That is, use $a = 10$. Then you can use a calculator's built-in base 10 logarithms.

$$x = \log_5 68 \Rightarrow x = \frac{\log 68}{\log 5}$$

$x \approx 2.62$

Evaluate each logarithmic expression to the nearest hundredth.

1. $\log_6 18$ ____________

2. $\log_5 100$ ____________

3. $\log_2 400$ ____________

4. $\log_8 512$ ____________

5. $\log_{10} 215$ ____________

6. $\log_{\frac{1}{2}} 24$ ____________

7. $\log_{13} 110$ ____________

8. $\log_{2.5} 76$ ____________

9. $\log_{\frac{1}{16}} 329$ ____________

◆Skill O Using the inverse functions $f(x) = e^x$ and $g(x) = \ln x$ to solve equations

Recall $\ln e$ is the base e logarithm of x. Therefore, $\ln e = 1$, just like $\log 10 = 1$ (base 10).

◆ **Example 1**

Simplify each expression.

a. $e^{\ln 4}$ **b.** $\ln e^2$

◆ **Solution**

a. Since $y = e^x$ and $y = \ln x$ are inverse functions, $e^{\ln x} = x$. So, $e^{\ln 4} = 4$.

b. Because of inverse functions, $\ln e^x = x$. So $\ln e^2 = 2$.

◆ **Example 2**

Solve for x.

a. $2e^{2x+1} = 60$ **b.** $\ln x = 3.2$

◆ **Solution**

a.
$$2e^{2x+1} = 60$$
$$e^{2x+1} = 30$$
$$\ln e^{2x+1} = \ln 30$$
$$2x + 1 = \ln 30$$
$$x = \frac{\ln 30 - 1}{2}$$
$$x \approx 1.20$$

b.
$$\ln x = 3.2$$
$$e^{\ln x} = e^{3.2}$$
$$x \approx e^{3.2}$$
$$x \approx 24.53$$

Simplify each expression.

1. $e^{\ln 4}$ ______

2. $e^{\ln 15}$ ______

3. $e^{2 \ln 3}$ ______

4. $\ln e^9$ ______

5. $\ln e^5$ ______

6. $5 \ln e^3$ ______

Solve each equation for x by using the natural logarithmic function.

7. $e^x = 34$ ______

8. $3e^x = 120$ ______

9. $e^x - 8 = 51$ ______

10. $\ln x = 2.5$ ______

11. $\ln(3x - 2) = 2.8$ ______

12. $\ln e^x = 5$ ______

◆Skill P Identifying parent functions in transformations

Recall Transformations of a function are indicated by the addition or subtraction of constants from the variable term or from the entire function or by multiplication or division of the variable term by a constant.

◆ **Example**

In the following equations, identify the parent function:

a. $y = -3|x - 2| + 5$ **b.** $y = -(x + 3)^2 - 4$

◆ **Solution**

a. Identify the additions, multiplications, subtractions, or divisions that occur. If the addition of 5 is removed, the equation becomes $y = -3|x - 2|$. If the multiplication by -3 is removed, the equation becomes $y = |x - 2|$. Finally, if the subtraction of 2 is removed, the equation becomes $y = |x|$. This is the absolute-value parent function.

b. Start with $y = -(x + 3)^2 - 4$ and remove the additions and subtractions, starting with the subtraction of 4 outside the parentheses. This leaves $y = -(x + 3)^2$. Then remove the negative sign preceding the parentheses, leaving $y = (x + 3)^2$. Finally, remove the addition of 3 within the parentheses, producing the function $y = x^2$. This is the quadratic parent function.

Identify the parent function for each of the following:

1. $y = -2|x + 1| - 4$ ____________________

2. $y = 3(x - 1)^2 - 2$ ____________________

3. $y = 3 \cdot 2^{-x} + 1$ ____________________

4. $y = -3(x + 2) - 4$ ____________________

5. $y = \frac{3}{x} + 2$ ____________________

6. $y = \frac{3}{x + 2}$ ____________________

7. $y = 3x^2 - 4$ ____________________

8. $y = -2(x - 1)^2$ ____________________

◆Skill Q Understanding the effect of order on combining transformations

Recall To determine the order of transformations to a function, reverse the order of operations. Addition or subtraction indicates a vertical translation; multiplication or division indicates a vertical stretch; addition or subtraction within parentheses or within absolute-value symbols indicates a horizontal translation.

◆ **Example**

Describe the various transformations included in the equation $y = 2|x - 1| + 3$.

◆ **Solution**

The first operation to consider is the addition of 3. This affects the parent function by translating it vertically 3 units up. The second operation, multiplication by 2, stretches the translated function by a factor of 2. The third operation, subtraction of 1, translates the stretched function horizontally 1 unit to the right. Thus, the parent function, $y = |x|$, has been shifted 1 unit to the right, stretched by a factor of 2, and then shifted 3 units up.

Describe the transformations of the parent functions included in each equation.

1. $y = -3|x + 2| - 3$ ______________________________

____

____

2. $y = 2(x - 3)^2 + 1$ ______________________________

____

____

3. $y = 4|x - 1| + 2$ ______________________________

____

____

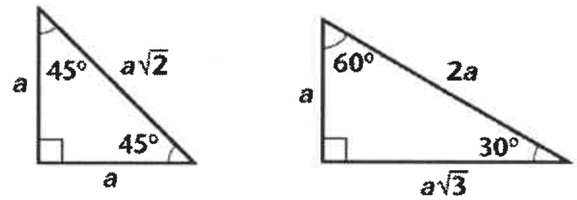
4. $y = 4 \cdot 2^x - 2$ ______________________________

____

____

◆Skill S Solving 45-45-90 and 30-60-90 triangles

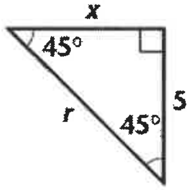
Recall



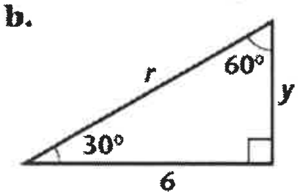
◆ **Example**

Find the lengths of the other 2 sides in each right triangle.

a.



b.



◆ **Solution**

a. $a = 5$

$x = a = 5$

$r = a\sqrt{2} = 5\sqrt{2}$

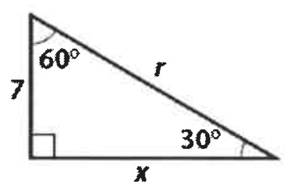
b. $a\sqrt{3} = 6 \rightarrow a = \frac{6}{\sqrt{3}}$ (or $2\sqrt{3}$)

$y = a = \frac{6}{\sqrt{3}}$ (or $2\sqrt{3}$)

$r = 2a = \frac{12}{\sqrt{3}}$ (or $4\sqrt{3}$)

Find the missing side lengths in each right triangle.

1.



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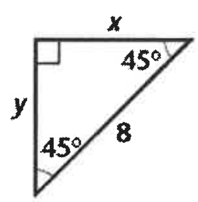
2.



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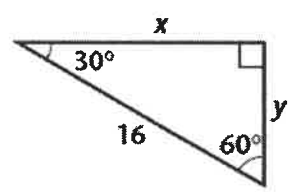
3.



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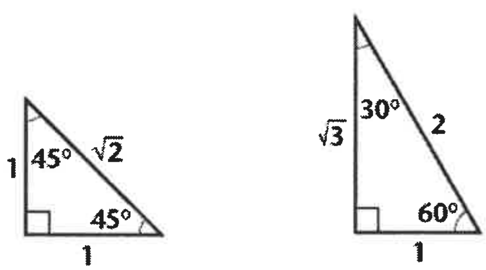
◆Skill T Finding exact values of the trigonometric functions for an angle whose reference angle is 30°, 45°, or 60°

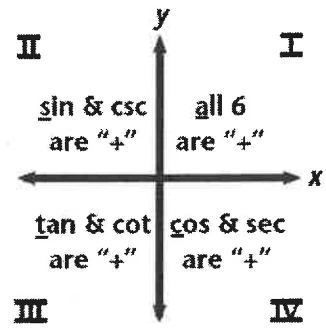
Recall



θ	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

A **reference angle** is the positive acute angle between the terminal side of a given angle and the x-axis.

One mnemonic for remembering which functions are **positive** in each quadrant is "All students take calculus."



◆ **Example**

Find each exact value.

a. sin 315° **b.** cos 240° **c.** tan 210°

◆ **Solution**

a. 315° is in Quadrant IV, where sine is negative. The reference angle is 45°.
$\sin 315° = -\sin 45°$
$= -\frac{1}{\sqrt{2}}$

b. 240° is in Quadrant III, where cosine is negative. The reference angle is 60°.
$\cos 240° = -\cos 60°$
$= -\frac{1}{2}$

c. 210° is in Quadrant III, where tangent is positive. The reference angle is 30°.
$\tan 210° = \tan 30°$
$= \frac{1}{\sqrt{3}}$

Find each trigonometric value. Give exact answers.

1. sin 120° ________ **2.** cos 330° ________ **3.** tan 225° ________ **4.** cos 150° ________

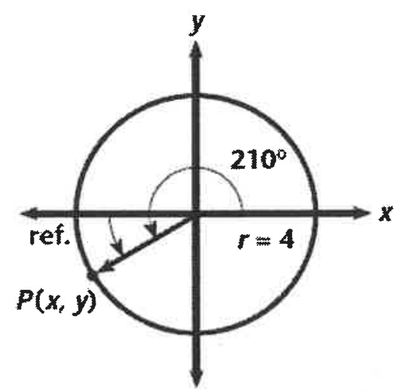
5. sin 240° ________ **6.** sin 150° ________ **7.** tan 315° ________ **8.** cos 225° ________

◆ **Skill U** Finding the coordinates of a point P on a circle

Recall If $P(x, y)$ lies at the intersection of the terminal side of θ in standard position and a circle centered at the origin with radius r, then $P(x, y) = P(r\cos\theta, r\sin\theta)$.

◆ **Example**

Find the coordinates of point P shown in the figure at right.



◆ **Solution**

$\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$ and

$\sin 210° = -\sin 30° = -\frac{1}{2}$

$r\cos\theta = 4\left(-\frac{\sqrt{3}}{2}\right)$ and $r\sin\theta = 4\left(-\frac{1}{2}\right)$

The coordinates of point P are $(-2\sqrt{3}, -2)$.

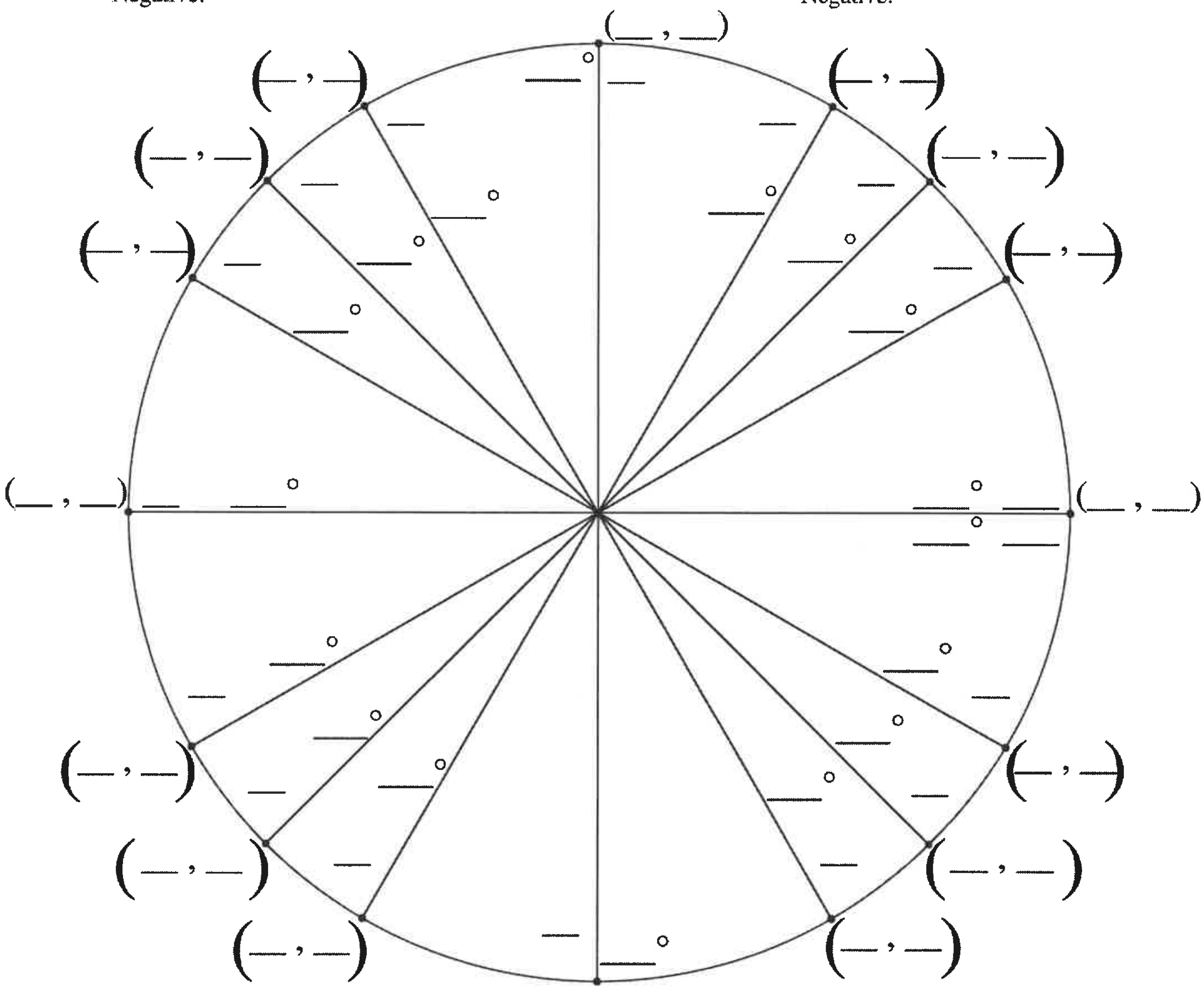
Point P is located at the intersection of a circle centered at the origin with a radius of r and the terminal side of angle θ in standard position. Find the exact coordinates of point P.

1. $\theta = 135°, r = 6$ ________ **2.** $\theta = 30°, r = 10$ ________ **3.** $\theta = 300°, r = 12$ ________

Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:



Positive:
Negative:

Positive:
Negative:

◆Skill V Finding exact values for the trigonometric functions of an angle measured in radians

◆ **Example**

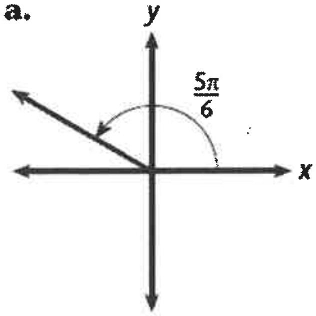
Give the exact value of each expression where the angle measures are in radians.

a. $\cos \frac{5\pi}{6}$ **b.** $\tan \frac{4\pi}{3}$ **c.** $\sin\left(-\frac{3\pi}{2}\right)$

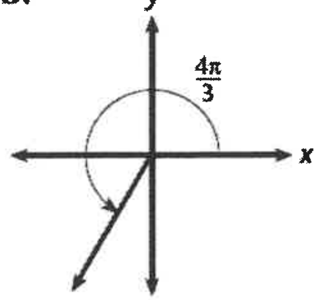
◆ **Solution**

Use reference angles.

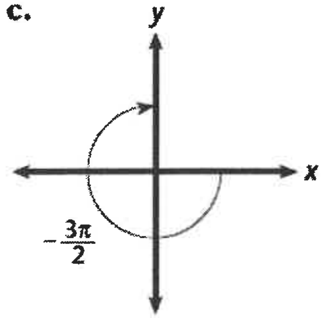
a.



b.



c.



$\frac{5\pi}{6}$ radians = 150°

$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\frac{4\pi}{3}$ radians = 240°

$\tan 240° = \tan 60° = \sqrt{3}$

$-\frac{3\pi}{2}$ radians = −270°

$\sin(-270°) = \sin 90° = 1$

Evaluate each expression. Give exact answers.

1. $\sin \frac{3\pi}{4}$ ____________

2. $\cos \frac{2\pi}{3}$ ____________

3. $\tan \frac{5\pi}{6}$ ____________

4. $\cos\left(-\frac{7\pi}{6}\right)$ ____________

5. $\tan\left(-\frac{\pi}{4}\right)$ ____________

6. $\sin \pi$ ____________

◆Skill W Converting between degree and radian measure

Recall π radians = 180°, so $\frac{\text{degree measure}}{180} = \frac{\text{radian measure}}{\pi}$.

◆ **Example 1**

Convert the following to radian measure.

a. 50° **b.** −150°

◆ **Solution**

a.
$$\frac{50^\circ}{180^\circ} = \frac{x}{\pi}$$
$$180x = 50\pi$$
$$x = \frac{50\pi}{180}$$
$$= \frac{5}{18}\pi$$
$$50^\circ = \frac{5}{18}\pi \text{ radians}$$

b.
$$-\frac{150^\circ}{180^\circ} = \frac{x}{\pi}$$
$$180x = -150\pi$$
$$x = -\frac{150\pi}{180}$$
$$= -\frac{5}{6}\pi$$
$$-150^\circ = -\frac{5}{6}\pi \text{ radians}$$

◆ **Example 2**

Convert the following to degree measure.

a. $\frac{2}{3}\pi$ radians **b.** $-\frac{7}{4}\pi$ radians

◆ **Solution**

a.
$$\frac{x}{180^\circ} = \frac{\frac{2}{3}\pi}{\pi}$$
$$\pi x = \frac{2}{3}\pi(180^\circ)$$
$$x = 120^\circ$$
$$\frac{2}{3}\pi \text{ radians} = 120^\circ$$

b.
$$\frac{x}{180^\circ} = \frac{-\frac{7}{4}\pi}{\pi}$$
$$\pi x = -\frac{7}{4}\pi(180^\circ)$$
$$x = -315^\circ$$
$$-\frac{7}{4}\pi \text{ radians} = -315^\circ$$

Convert the following degree measures to radian measures. Give exact answers.

1. 270° ________ **2.** 45° ________ **3.** 225° ________

4. 210° ________ **5.** −90° ________ **6.** −300° ________

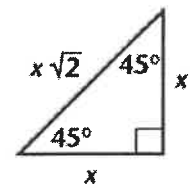
Convert each of the following radian measures to degree measures.

7. $\frac{\pi}{4}$ ________ **8.** $\frac{3\pi}{2}$ ________ **9.** $\frac{5\pi}{6}$ ________

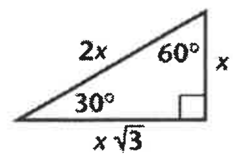
10. $\frac{5\pi}{3}$ ________ **11.** -3π ________ **12.** $-\frac{11\pi}{6}$ ________

◆Skill X Using the 45-45-90 Triangle Theorem and the 30-60-90 Triangle Theorem

Recall A 45-45-90 triangle is a right triangle in which both acute angles have measure 45°. In a 45-45-90 triangle, if the length of each leg is x, the length of the hypotenuse is $x\sqrt{2}$. If the length of the hypotenuse is x, then the length of each leg is $\frac{x}{\sqrt{2}}$ or $\frac{x\sqrt{2}}{2}$.



A 30-60-90 triangle is a right triangle in which the acute angles have measures of 30° and 60°. The shorter leg of the triangle is opposite the 30° angle and the longer leg is opposite the 60° angle. In a 30-60-90 triangle, if the length of the shorter leg is x, then the length of the longer leg is $x\sqrt{3}$, and the length of the hypotenuse is $2x$.



◆ **Example**

a. The legs of a 45-45-90 triangle are each 15 centimeters long. Find the length, h, of the hypotenuse to the nearest tenth of a centimeter.

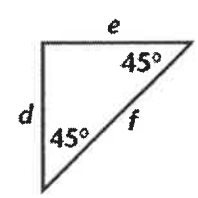
b. The longer leg of a 30-60-90 triangle is 5 inches long. Find the length, h, of the hypotenuse in simplest radical form.

◆ **Solution**

a. Refer to the 45-45-90 triangle in the figure above. Since $x = 15$, $h = x\sqrt{2} = 15\sqrt{2} \approx 21.2$ centimeters.

b. Refer to the 30-6-90 triangle in the figure above. Since $x\sqrt{3} = 5$, $x = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ and $h = 2x = \frac{10\sqrt{3}}{3}$ inches.

The length of one side of a 45-45-90 triangle is given. Find the lengths of the other two sides in simplest radical form.



1. $d = 13$, $e =$ ____________

$f =$ ____________

2. $d = 4$, $e =$ ____________

$f =$ ____________

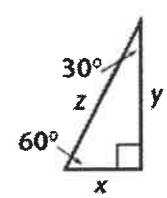
3. $e = 4.5$, $f =$ ____________

$d =$ ____________

4. $f = 9\sqrt{2}$, $d =$ ____________

$e =$ ____________

The length of one side of a 30-60-90 triangle is given. Find the lengths of the other two sides in simplest radical form.



5. $x = 4$, $y =$ ____________

$z =$ ____________

6. $x = 3\sqrt{3}$, $z =$ ____________

$y =$ ____________

7. $y = 7\sqrt{3}$, $x =$ ____________

$z =$ ____________

8. $y = 18$, $z =$ ____________

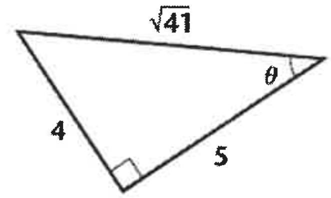
$x =$ ____________

◆Skill Y Finding the trigonometric functions of an acute angle

Recall The hypotenuse is the longest side in a right triangle and is opposite the right angle.

◆ **Example**

Refer to the triangle shown at right and give values for $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$.



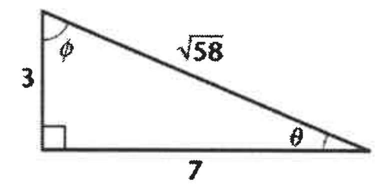
◆ **Solution**

The hypotenuse (hyp.) has a length of $\sqrt{41}$.
The leg opposite (opp.) θ has a length of 4.
The leg adjacent (adj.) to θ has a length of 5.

$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{4}{\sqrt{41}}$ $\quad\csc\theta = \frac{\text{hyp.}}{\text{opp.}} = \frac{\sqrt{41}}{4}$ $\quad\cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{5}{\sqrt{41}}$

$\sec\theta = \frac{\text{hyp.}}{\text{adj.}} = \frac{\sqrt{41}}{5}$ $\quad\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{4}{5}$ $\quad\cot\theta = \frac{\text{adj.}}{\text{opp.}} = \frac{5}{4}$

Refer to the triangle at right to find each value. Give exact answers.



1. $\sin\theta$ ____________ **2.** $\cos\theta$ ____________

3. $\tan\theta$ ____________ **4.** $\csc\theta$ ____________

5. $\sec\theta$ ____________ **6.** $\cot\theta$ ____________ **7.** $\sin\phi$ ____________

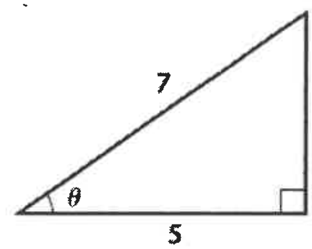
8. $\cos\phi$ ____________ **9.** $\tan\phi$ ____________ **10.** $\csc\phi$ ____________

◆Skill Z Using inverse trigonometric functions to find the measure of an acute angle

Recall The statements $\tan \theta = \frac{5}{7}$ and $\theta = \tan^{-1}\left(\frac{5}{7}\right)$ are equivalent.

◆ **Example**

In the triangle shown at right, find the measure of θ to the nearest whole degree.



◆ **Solution**

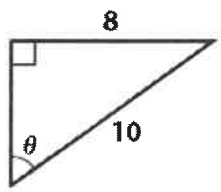
Since the **hypotenuse** and the side **adjacent** to θ are given, use the cosine function.

$$\cos \theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{5}{7}$$

$$\theta = \cos^{-1}\left(\frac{5}{7}\right) \approx 44°$$ *Use your calculator in **degree** mode.*

Find θ to the nearest degree in each triangle.

1.



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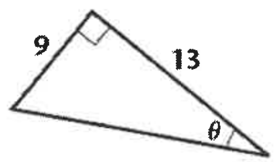
2.



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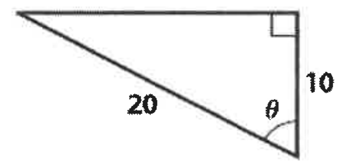
3.



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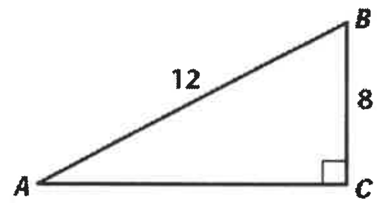
◆Skill AA Solving a right triangle

Recall *Solving a triangle* means to use given measures to find the unknown measures of the other sides and angles of the triangle.

◆ **Example 1**

Solve the triangle shown at right.



◆ **Solution**

Using the hypotenuse and side opposite $\angle A$,

$\sin \angle A = \frac{8}{12}$

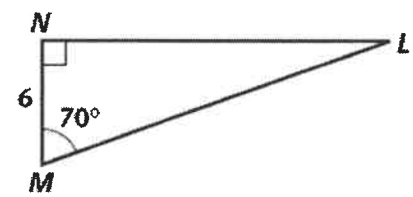
$m\angle A = \sin^{-1}\left(\frac{8}{12}\right) \approx 42°$ *Round to the nearest whole degree.*

$$m\angle A + m\angle B = 90°$$
$$42° + m\angle B = 90°$$
$$m\angle B \approx 48°$$

$$8^2 + (AC)^2 = 12^2$$
$$AC = \sqrt{12^2 - 8^2}$$
$$AC \approx 8.9$$

◆ **Example 2**

Solve the triangle shown at right.



◆ **Solution**

Using the side adjacent to $\angle M$,

$\cos 70° = \frac{6}{LM}$, where LM is the hypotenuse.

$LM = \frac{6}{\cos 70°} \approx 17.5$

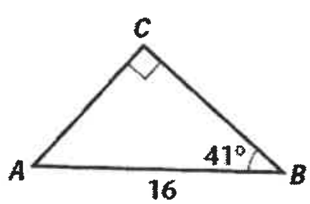
$(LN)^2 + (MN)^2 = (LM)^2$

$LN \approx \sqrt{17.5^2 - 6^2} \approx 16.4$

$$m\angle L + m\angle M = 90°$$
$$m\angle L + 70° = 90°$$
$$m\angle L = 20°$$

Solve each triangle. Round each angle measure to the nearest degree and each side length to the nearest tenth.

1.



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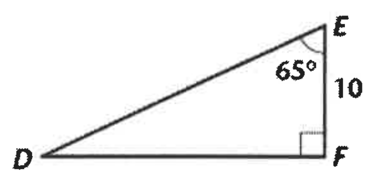
2.



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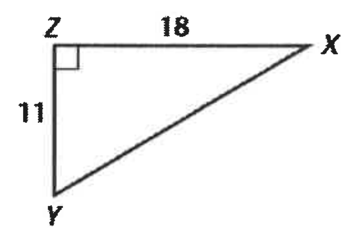
3.



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◆Skill BB Evaluating inverse trigonometric relations and functions

Recall The domain and range of a function become the range and domain respectively, of the inverse.

◆ **Example 1**

Find each value. Give answers in degrees and radians.

a. $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **b.** $\text{Cos}^{-1}\left(-\frac{1}{2}\right)$ **c.** $\text{Tan}^{-1}(-1)$

◆ **Solution**

a. Since $\sin 60° = \frac{\sqrt{3}}{2}$, then $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$ or $\frac{\pi}{3}$ radians.

Notice that although other angles have a sine of $\frac{\sqrt{3}}{2}$, you must choose an angle that is between $-90°$ and $90°$ in order to have a value in the appropriate range.

b. Since $\cos 120° = -\frac{1}{2}$, then $\text{Cos}^{-1}\left(-\frac{1}{2}\right) = 120°$ or $\frac{2\pi}{3}$ radians.

c. Since $\tan(-45°) = -1$, then $\text{Tan}^{-1}(-1) = -45°$ or $-\frac{\pi}{4}$ radians.

◆ **Example 2**

Evaluate each expression.

a. $\sin\left(\text{Cos}^{-1}\left(\frac{1}{2}\right)\right)$ **b.** $\text{Tan}^{-1}(\sin 90°)$

◆ **Solution**

a. Begin inside the parentheses.

$\text{Cos}^{-1}\left(\frac{1}{2}\right) = 60°$

So, $\sin\left(\text{Cos}^{-1}\left(\frac{1}{2}\right)\right) = \sin 60° = \frac{\sqrt{3}}{2}$

b. $\sin 90° = 1$

Therefore, $\text{Tan}^{-1}(\sin 90°) = \text{Tan}^{-1}(1)$

$= 45°$ or $\frac{\pi}{4}$ radians

Find each value. Give answers in degrees and in radians. (It may be helpful to review what you learned about 30°-, 45°-, and 60°-angles.)

1. $\text{Sin}^{-1}\left(\frac{1}{2}\right)$ ____________ **2.** $\text{Cos}^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ ____________ **3.** $\text{Tan}^{-1}(\sqrt{3})$ ____________

4. $\text{Sin}^{-1}(-1)$ ____________ **5.** $\text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ ____________ **6.** $\text{Tan}^{-1}(-1)$ ____________

Evaluate each composite trigonometric expression.

7. $\tan\left(\text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ ____________ **8.** $\cos\left(\text{Sin}^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$ ____________ **9.** $\sin\left(\text{Tan}^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right)$ ____________

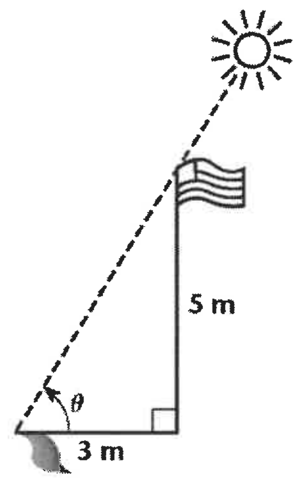
10. $\text{Sin}^{-1}(\cos 0°)$ ____________ **11.** $\text{Tan}^{-1}(\sin 0°)$ ____________ **12.** $\text{Sin}^{-1}(\sin 90°)$ ____________

◆Skill CC Applying inverse trigonometric functions

Recall $\sin \theta = \frac{\text{opp.}}{\text{hyp.}}$ $\cos \theta = \frac{\text{adj.}}{\text{hyp.}}$ $\tan \theta = \frac{\text{opp.}}{\text{adj.}}$

◆ **Example**

At a certain time of the day, the 5 meter flagpole shown at right casts a shadow that is 3 meters long. What is the angle of elevation of the sun at this time?



◆ **Solution**

Since 3 meters is the length of the side **adjacent** to θ and 5 meters is the length of the side **opposite** θ, use the tangent function.

$\tan \theta = \frac{5}{3}$

$\theta = \tan^{-1}\left(\frac{5}{3}\right)$

This last equation states that θ is the angle that has a tangent of $\frac{5}{3}$.

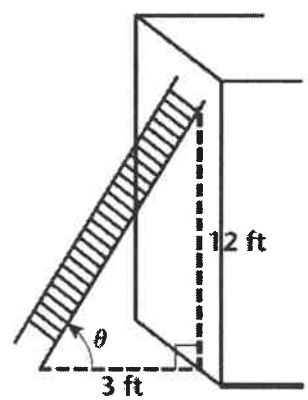
$\theta \approx 59°$ *Use calculator in **degree** mode.*

Find the measure of each angle to the nearest whole degree.

1. Find the measure of the smallest angle in a right triangle with sides of 3, 4, and 5 centimeters. ____________

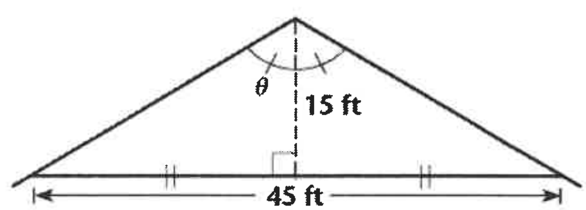
2. What is the angle between the bottom of the ladder and the ground as shown at right?

____



3. Find the angle at the peak of the roof as shown at right.

____



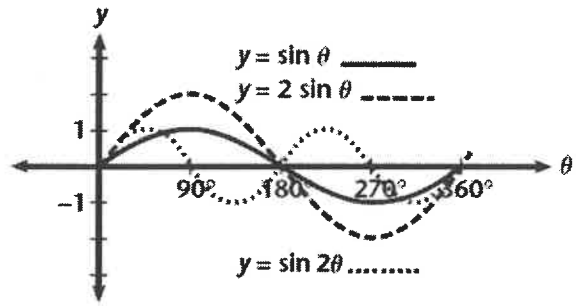
4. The hypotenuse of a right triangle is 3 times as long as the shorter leg. Find the measure of the angle between the shorter leg and the hypotenuse. ____________

◆Skill DD Graphing functions of the form $y = a \sin b\theta$, $y = a \cos b\theta$, and $y = a \tan b\theta$

Recall The sine and cosine are periodic functions with a period of 360° or 2π radians. The tangent function has a period of 180° or π radians.

◆ **Example 1**
Graph $y = \sin \theta$, $y = 2 \sin \theta$, and $y = \sin 2\theta$ on the same set of axes.

◆ **Solution**
The graph of $y = a \sin b\theta$ has an amplitude (height above x-axis) of $|a|$ and period of $\frac{360°}{b}$.

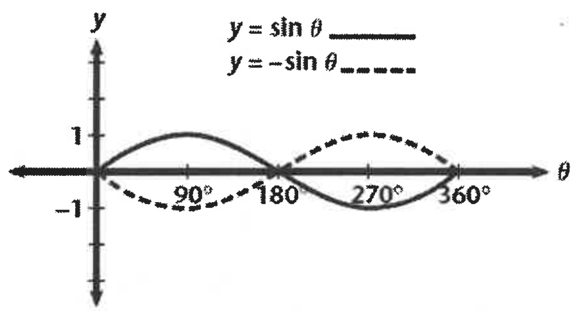


function	amplitude	period
$y = \sin \theta$	1	360°
$y = 2 \sin \theta$	2	360°
$y = \sin 2\theta$	1	180°

Check your results with a graphics calculator.

◆ **Example 2**
Graph $y = \sin \theta$ and $y = -\sin \theta$ on the same set of axes.

◆ **Solution**
Notice that the graph of $y = -\sin \theta$ is the reflection of $y = \sin \theta$ across the (horizontal) θ-axis.



Sketch each pair of functions on the same set of axes. Use $0° \leq \theta \leq 360°$.

1. $y = \cos \theta$, $y = \frac{1}{2}\cos \theta$

2. $y = \cos \theta$, $y = \cos 3\theta$

3. $y = \tan \theta$, $y = -\tan \theta$

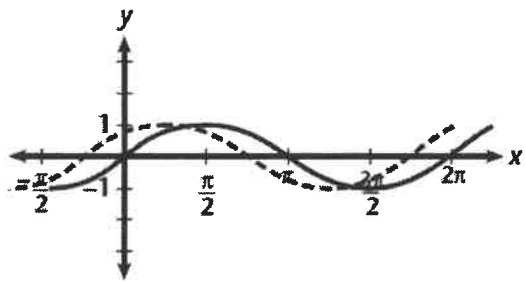
4. $y = \tan \theta$, $y = \tan \frac{1}{2}\theta$

◆Skill EE Graphing functions of the form $y = \sin(x - c) + d$, $y = \cos(x - c) + d$, and $y = \tan(x - c) + d$

Recall The graph of $y = \sin(x - c)$ is a **phase shift** (horizontal translation) of the graph of $y = \sin x$ to the **right** c units.
The graph of $y = \sin x + d$ is a **vertical shift** of the graph of $y = \sin x$ up d units.

◆ **Example 1**
Graph $y = \sin x$ and $y = \sin\left(x + \frac{\pi}{4}\right)$ on the same set of axes.

◆ **Solution**

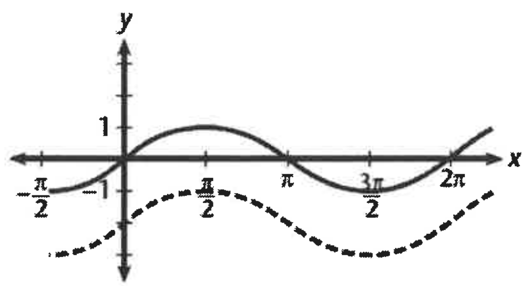


$y = \sin x$ ———
$y = \sin\left(x + \frac{\pi}{4}\right)$ - - - - -

phase shift of $\frac{\pi}{4}$ units to the left

◆ **Example 2**
Graph $y = \sin x$ and $y = \sin x - 2$ on the same set of axes.

◆ **Solution**

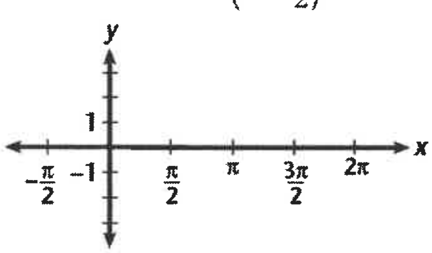


$y = \sin x$ ———
$y = \sin x - 2$ - - - - -

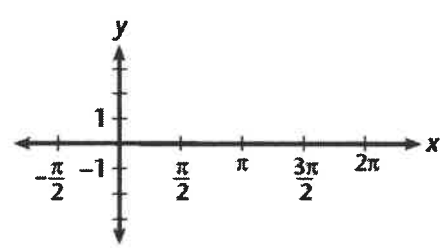
vertical shift of 2 units down

Sketch each pair of functions on the same set of axes. Use $-\frac{\pi}{2} \le x \le 2\pi$.

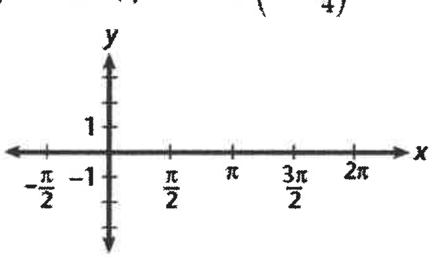
1. $y = \cos x$, $y = \cos\left(x - \frac{\pi}{2}\right)$



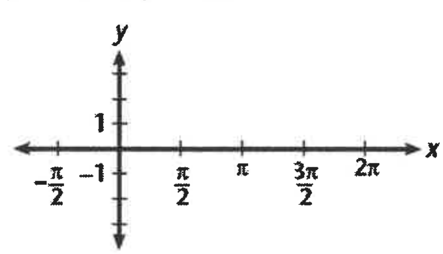
2. $y = \cos x$, $y = \cos x + 1$



3. $y = \tan x$, $y = -\tan\left(x + \frac{\pi}{4}\right)$



4. $y = \tan x$, $y = \tan x - 1$



◆Skill FF Verifying the fundamental trigonometric identities

Recall $\csc \theta = \frac{1}{\sin \theta}$, $\sec \theta = \frac{1}{\cos \theta}$, $\cot \theta = \frac{1}{\tan \theta}$

Also, $\sin^2 \theta = (\sin \theta)(\sin \theta)$ or $(\sin \theta)^2$, which is the form you will need to enter into your graphics calculator.

◆ **Example**

Use a calculator to verify that

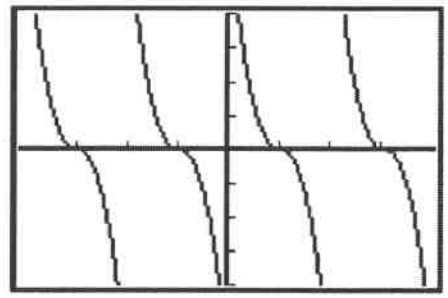
$\cot \theta = \frac{\cos \theta}{\sin \theta}$.

◆ **Solution**

Graph $y_1 = \frac{1}{\tan \theta}$ (for $\cot \theta$)

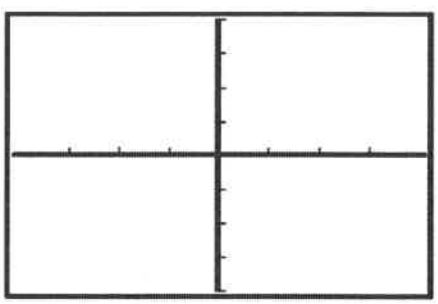
and $y_2 = \frac{\cos \theta}{\sin \theta}$.

The graphs match exactly.

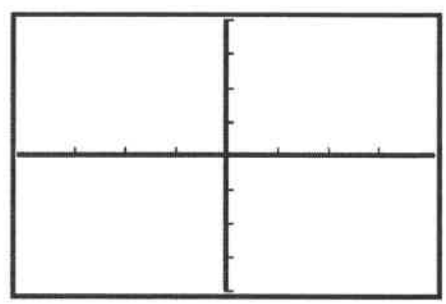


Verify each identity by graphing each side separately. Sketch the common graph.

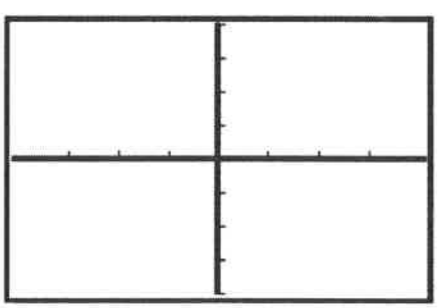
1. $\tan \theta = \frac{\sin \theta}{\cos \theta}$



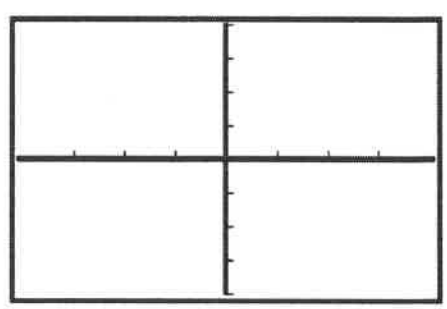
2. $\sin^2 \theta + \cos^2 \theta = 1$



3. $\tan^2 \theta + 1 = \sec^2 \theta$



4. $1 + \cot^2 \theta = \csc^2 \theta$



◆Skill GG Simplifying expressions by using basic trigonometric identities

Recall Since $\sin^2\theta + \cos^2\theta = 1$, then $\sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$.

◆ **Example 1**

Simplify $\csc\theta \tan\theta$ to $\sec\theta$.

◆ **Solution**

$\csc\theta\tan\theta = \frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}$ *fundamental identities*

$= \frac{1}{\cos\theta}$ $\frac{\sin\theta}{\sin\theta} = 1$

$= \sec\theta$ *fundamental identity*

◆ **Example 2**

Simplify $(\sec\theta - 1)(\sec\theta + 1)$ to $\tan^2\theta$.

◆ **Solution**

$(\sec\theta - 1)(\sec\theta + 1) = \sec^2\theta - 1$ $(a - b)(a + b) = a^2 - b^2$

$= \tan^2\theta + 1 - 1$ *fundamental identity*

$= \tan^2\theta$

For exercises 5–10, show on your own paper how the first expression simplifies to the second expression.

1. $\sin x \cot x$ to $\cos x$
2. $\sin x \sec x \cot x$ to 1
3. $\cos^2 x - \sin^2 x$ to $1 - 2\sin^2 x$
4. $(1 + \sin x)(1 - \sin x)$ to $\cos^2 x$
5. $\tan x + \cot x$ to $\sec x \csc x$
6. $(\cos x - \sin x)^2$ to $1 - 2\cos x \sin x$